RTCA Special Committee 186, Working Group 5

ADS-B UAT MOPS

Meeting #4

UAT Performance (Undetected Error Rates)

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SUMMARY

This paper addresses an action item (3-15) from meeting #3 to consider Reed Solomon codes for UAT that can provide undetected burst error rates of less than 10^{-8} without additional CRC coding. It is shown that to meet this requirement it would be necessary to use RS(29,17) and RS(47,33) codes for the short and long ADS-B messages, respectively. If, instead, the requirement were 10^{-7}, then codes RS(27,17) and RS(45,33) would be sufficient.

The goal of this effort is to find error correction codes for UAT that will provide undetected error probabilities (per burst) that are less than 10^{-8}. The possible codes for long ADS-B messages are RS(45,33) and RS(47,33). For the short ADS-B messages they are RS(27,17) and RS(29,17). The sole candidate for the up link message is 6xRS(85,65). It is assumed that none of these formats includes CRC. The asymptotic values for the undetected error rates (achieved when the channel bit error rates are 0.5) for these codes are given in the table.

Code	Maximum Undetected Word Error Rate
RS(27,17)	7.21e-8
RS(29,17)	1.65e-9
RS(45,33)	2.83e-8
RS(47,33)	8.50e-10
RS(85,65)	2.49e-12

Table 1. Maximum Undetected RS Word Error Rates

These values were calculated using a formula equivalent to that provided in working paper UAT-WP2-03.

The performance of these codes as functions of channel bit error rate are shown in figures 1 through 5. These graphs show total word error rate together with undetected word error rate.

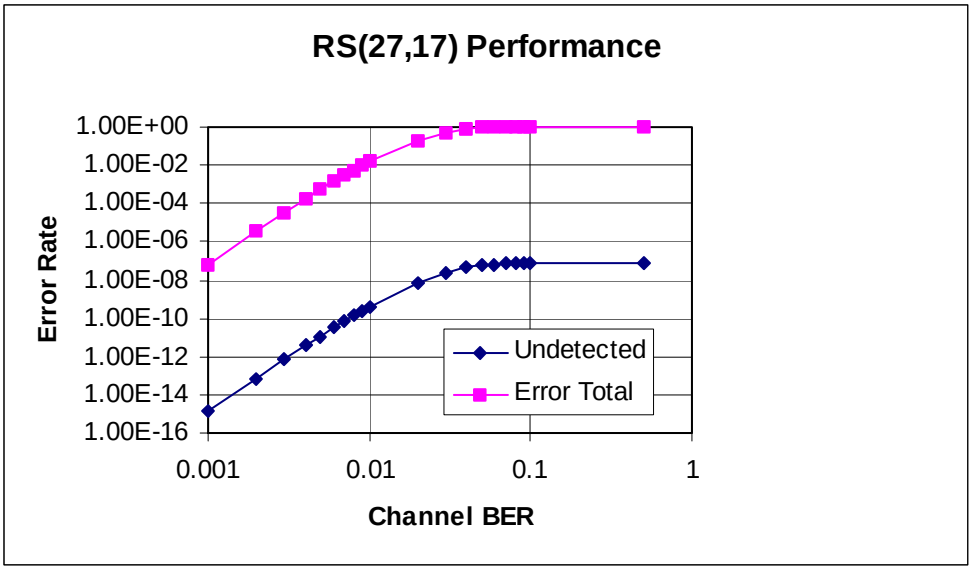

Figure 1. RS(27,17) Performance

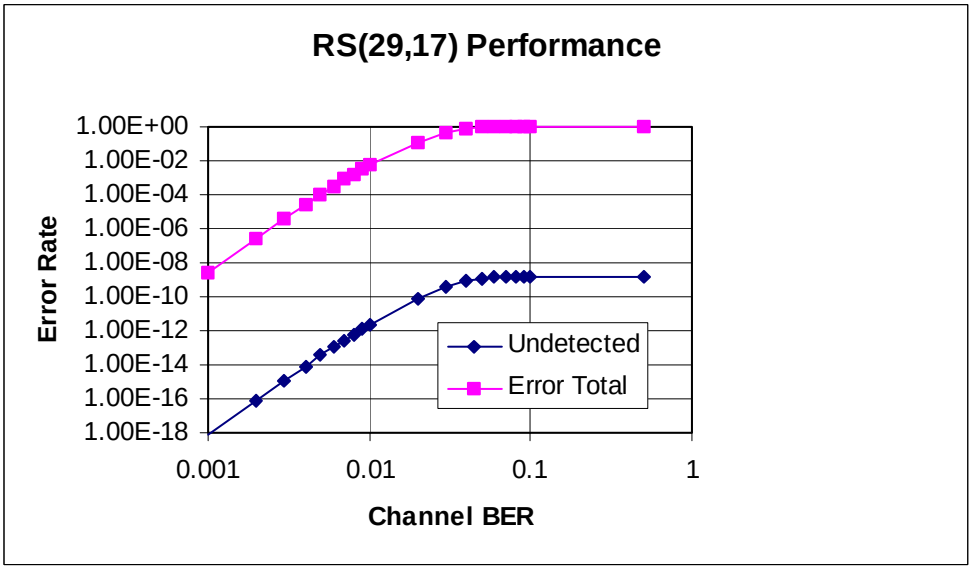

Figure 2. RS(29,17) Performance

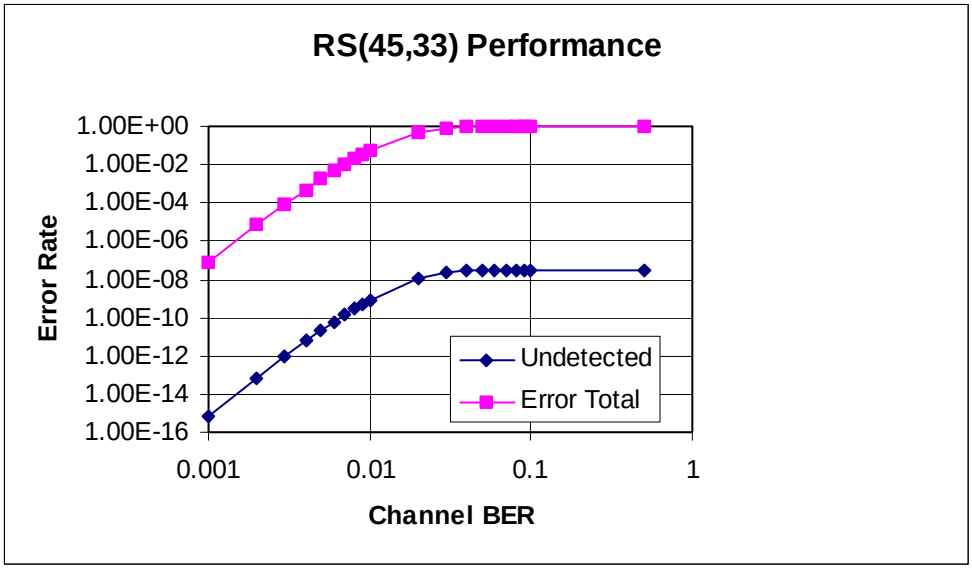

Figure 3. RS(45,33) Performance

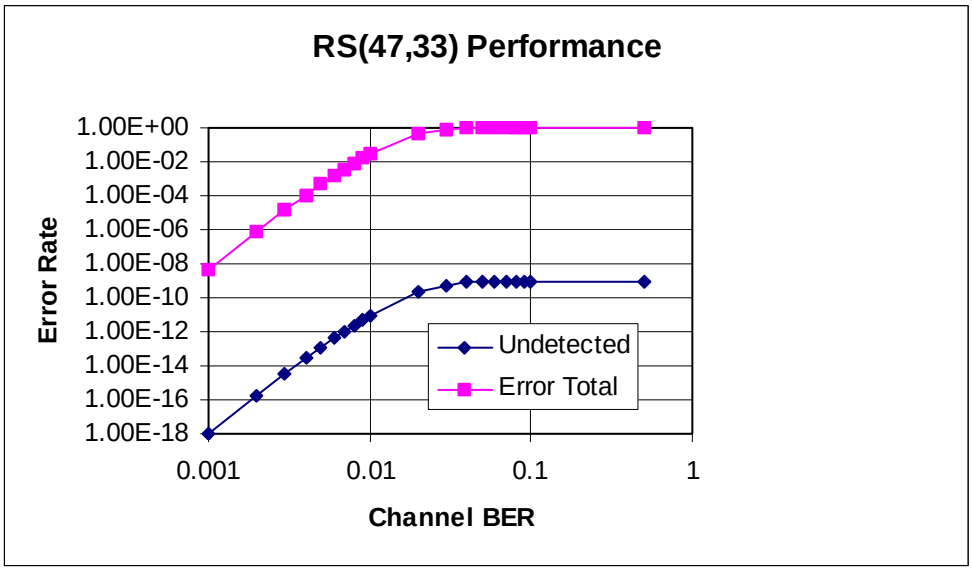

Figure 4. RS(47,33) Performance

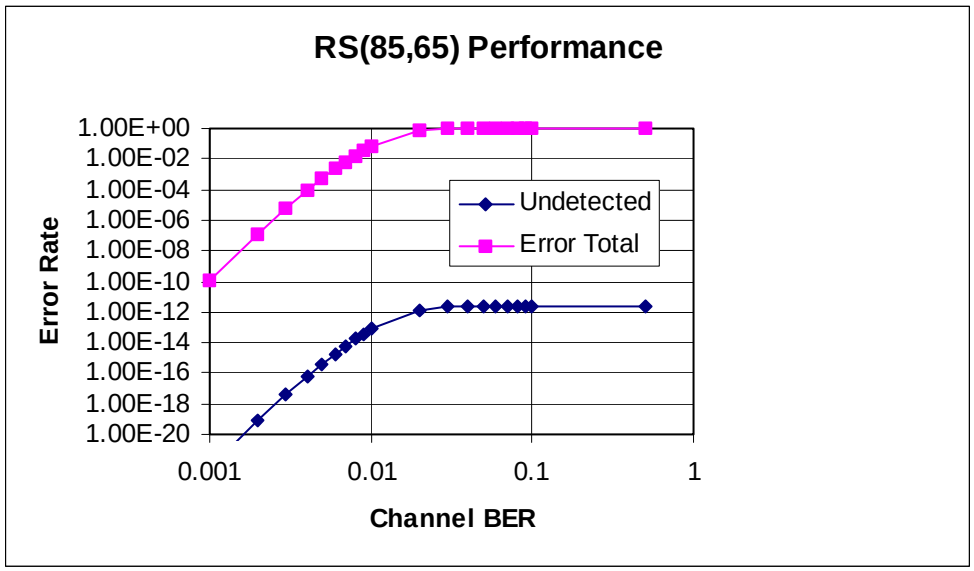

Figure 5. RS(85,65) Performance

Note that this last graph shows the performance of a single RS(85,65) word. The performance of an entire burst, consisting of six words is given by

$$P_{Uburst} = (1 - P_E + P_U)^6 - (1 - P_E)^6$$

and

$$P_{Eburst} = 1 - (1 - P_E)^6$$

P_E is the total word error rate and P_U is the undetected word error rate. A graph of the undetected burst error rate versus the channel BER is shown in figure 6. This figure indicates that the maximum undetected error rate is about 5.5e-13, which occurs when the channel BER is about 0.0125. To see why there is a maximum, we can make the following approximation

$$P_{UBurst} \approx 6P_U(1 - P_E)^5.$$

The P_U term is small at low BER and the $(1 - P_E)^5$ term is small at high BER (because P_E is nearly 1 in that case).

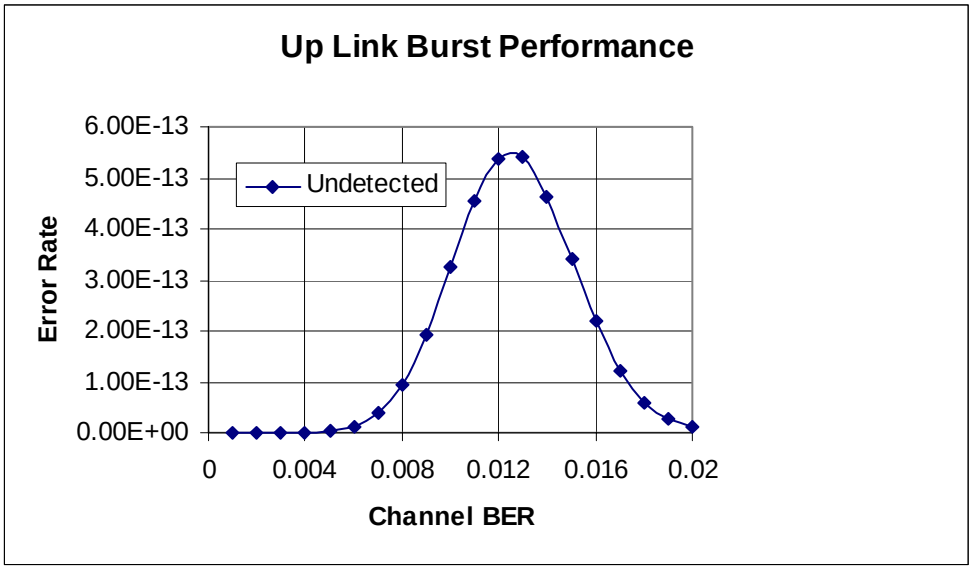

Figure 6. Up Link Undetected Burst Error Rate

The bottom line is that if we choose the codes RS(29,17), RS(47,33), and 6xRS(85,65) all the undetected error rates will be comfortably less than 10^{-8}. On the other hand, if the undetected error probability requirement were set at 10^{-7}, then RS(27,17) and RS(45,33) would be sufficient.